ROSETTA Implementation in SysML v2 Revisited

Follow up to the Mathsig Presentation to the SE DSIG, March 2023

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ROSETTA: Relational Oriented Systems Engineering Technology Trade-off and Analysis [1]
SysML: Systems Modeling Language
SE: Systems Engineering
DSIG: Domain Special Interest Group in the Object Management Group (OMG)
Mathsig: OMG Mathematical Formalism DSIG

mathsig/2024-06-02

Key findings from the March 2023 SE DSIG presentation [1]

- The aim of the presentation was to demonstrate that the language capabilities of SysML v2 can implement ROSETTA and constraint driven design [1, 2].
- Initial findings included:
    - UPR 1.0 (UML Profile for ROSETTA) [2] could be implemented in SysML v2 using artefacts to include Constraint Definition and Part Definition Textual Syntax.
    - Basic mathematical concepts of engineering design could also be implemented but it was not clear how system ***relations between relations*** could be.

ROSETTA: Relational Oriented Systems Engineering Technology Trade-off and Analysis [1]
SysML: Systems Modeling Language
SE DSIG: Systems Engineering Domain Special Interest Group

Model Based Systems Engineering (MBSE) Context

- *A key challenge in MBSE is to specify models that are mathematical and executable.*
- ROSETTA frameworks offer a mathematical relational viewpoint on MBSE [2, 3].
- The Mathematical Formalism DSIG is working on foundational formalisms that underlie MBSE and can be expressed via OMG model-based standards [1].
- In June 2018, UPR 1.0 [2] was adopted (the UML Profile for ROSETTA). Constraints were difficult to model in SysML v1 but now can be modelled in SysML v2 [1, 4].
- In March 2023, the 'Mathsig' presented a *demonstration* of how constraints in UPR 1.0 can be implemented in SysML v2 [1] but did not use the methods of ROSETTA.
- The *demonstration* was offered as an argument for adoption of SysML v2 in 2023.

This presentation is concerned with how the relational viewpoint of ROSETTA can be used to enhance SysML v2 expressiveness for MBSE.

Advancements to meet the MBSE Challenge

- A multi-objective multi-attribute *radar system safety analysis problem* was the basis for assessing SysML v2 language capabilities in the March 2023 presentation to the SE DSIG.
- The Mathsig presentation to the SE DSIG in 2023 *demonstrated* that issues of implementing UPR 1.0 in SysML v1 were resolved by SysML v2 … however,

  There were issues of modelling and analysis of system interrelations.

  How to model *relations between relations* (interrelations) using SysML v2 is not clear.

  Interrelations can also occur in data sets (e.g., from design of experiments).

- SysML is not an execution language but should support the capture of system interrelations if the system models are to support analysis in a tool that can execute the models.
- The MBSE challenge to specify mathematical models that are executable must be met in the continued evolution and commercialisation of OMG languages and tools.

The ROSETTA framework for radar system design and safety analysis will model the interrelations and offer a structured MBSE solution.

Topics of the Presentation

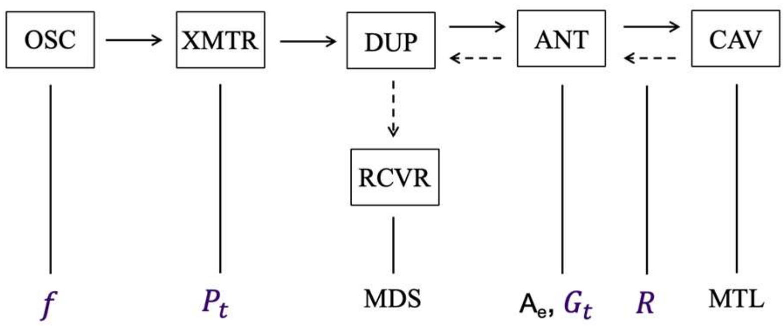
- Engineering viewpoint of the *radar system design and safety analysis*
- Implementation in SysML v2 and ROSETTA; elementary solution
- Interrelation (relations between relations) implementation in ROSETTA
- Conclusions: Implementation of ROSETTA in SysML v2?

This presentation is about modelling and analysis for engineering design.

Multi-objective, muti-attribute system design*

Engineering viewpoint

OSC: oscillator
XMTR: transmitter
DUP: duplexer
RCVR receiver
ANT: antenna
CAV: cooperative air vehicle
f: frequency
P_t: power
MDS: min detectable signal
A_e: effective aperture
G: transmission gain
R: range
MTL: min triggering level



*The problem is to optimise the Power Aperture ($P_t A_e$) objective ***subject to a safety constraint objective***
- Power Aperture Product is analogous to 'horsepower' for the radar (and determines detection range)
- Safety is measured by the Power Density ($P_t G_t/4\pi d^2$) at a specified perimeter (d) around the radar

*Adapted from the Loughborough University WS66 System Design MSc module and textbook [5].

Radar design and safety analysis case study (1 of 2)

- Purpose:

  provide a practical real world engineering reference problem that stresses interrelationship modelling & analysis

- The objective variables are

  $z_1 = P_t A_e$ power aperture product (→ radar performance)

  $z_2 = P_t G_t / 4\pi d^2$ power density at perimeter (→ radar safety)

- Ordinary constraints on variables as per UPR 1.0 are

  Performance $R(z_1)$: $r \leq z_1$; also, is ***constrained by safety***

  Safety $S(z_2)$: $z_2 \leq s$ *(e.g. 50W/m²)*

Formulae can be expressed in SysML v2.

Objectives:

$f(P_t, A_e) = z_1 = P_t A_e$

$h(P_t, G_t) = z_2 = P_t G_t / 4\pi d^2$

Interrelations:

$g(P_t, A_e) = (P_t, G(A_e))$

$G(A_e) = 4\pi A_e / \lambda^2$

- Modelling of the objectives and constraints is a natural application of SysML v2:

  Formulae can be stored in Part Definition diagrams

  Constraints can be stored in requirements Constraint Definition diagrams

- *It is straight forward to model the problem in SysML v2 if the objectives are independent.*

Radar design and safety analysis case study (2 of 2)

- Defining a specific safety perimeter d results in an ordinary constraint on $z_1 = P_t A_e$ e.g.,

  For $\lambda d = 1m^2$ and $\lambda = 0.3m$, 50 W/m² → 50Wm²

  (the safety constraint implies a performance constraint)*

- The performance constraint $R(z_1)$: $r \leq z_1$ then becomes $r \leq z_1 \leq 50Wm^2$, with *r* to be defined by further analysis.
- When the variables of the power aperture *radar system design space* (X_1) are given in decibels (dB), the problem is linearised.
- The constraint on power aperture, z_1, is *bounded by* $x_{11} + x_{12} = 17$ dBWsm i.e., $x_{11}\, x_{12} = 50Wm^2$.

Implied Constraint on Power Aperture For a Perimeter Satisfying λd = 1m²

The use of decibels (dB) linearises the variables. (50 Wm² → 17 dBWsm)

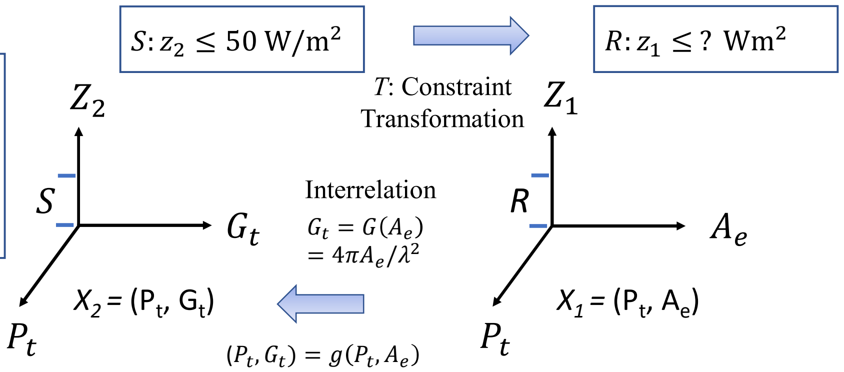
*Note: this was the relation between constraints used in the March 2023 presentation to the SE DSIG.

Engineering Representation of the Constraint Transformation

One 'value' of the transformation constrains power-aperture to: $z_1 \leq 50Wm^2$ (17dBWsm).

Algebraic Representation of Domain Knowledge

$f(P_t, A_e) = z_1 = P_t A_e$
$h(P_t, G_t) = z_2 = P_t G_t / 4\pi d^2$
$g(P_t, A_e) = (P_t, G(A_e))$
$G(A_e) = 4\pi A_e / \lambda^2$



At the safety perimeter of d meters, power density z_2 must be $\leq 50W/m^2$.

What is the general form of the constraint transformation $T: Z_1 \rightarrow Z_2$?

Transformation of requirements into technical views
ISO/IEC/IEEE 15288:2015 System Requirements Definition [6]

6.4.3.1 *The purpose of the System Requirements Definition process is to* ***transform**** *the stakeholder, user-oriented* ***view of desired capabilities into a technical view*** *of a solution …*

- In the radar case study, the user-oriented *view of radar safety* was represented by a constraint (power density ≤ 17 dBW/sm) which was ***transformed*** *into a mathematical model of the power aperture solution set* (i.e., a technical view),

  $P + A_e \leq 17$ (dBWsm) at the safety perimeter (d = 3.3m)

- This is an instance, i.e. value, of the transformation of requirements when the radar perimeter d is specified by λd = 1m² with λ = 0.3m.
- Transformation: $f(P, A_e) = P + A_e$ dBWsm is constrained by safety (17 dBW/sm)

**The radar case study is a demonstration of how mathematical formalisms that underlie MBSE can express the terms and concepts of systems engineering standards.*

Topics of the Presentation

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- ***Implementation in SysML v2 and ROSETTA; elementary solution***
- Interrelation (relations between relations) implementation in ROSETTA
- Conclusions: Implementation of ROSETTA in SysML v2?

Using ROSETTA for elementary system design is straight forward.

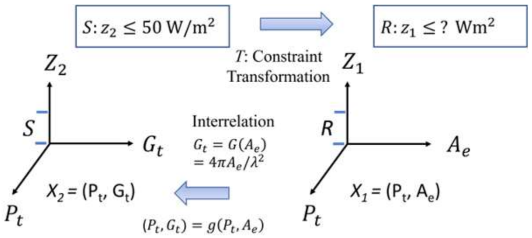
Snapshot of the system model in SysML v2 textual notation*

```
part radar:RadarSystem{
        attribute redefines safetyPerimeter = 3.3 [m];
        attribute powerAperture:> powerAperture = t. power + a.aperature;
        attribute powerDensity:> powerDensity = t.power*a.gain/(4*pi*radar.safetyPerimeter);
        part t.Transmitter{
                attribute power:> dBW;
        }
        part a.Antenna{
                attribute aperture:> dBsm;
                attribute gain:> gain = 4*pi*a.aperture/o.s.wavelength^2;
        }
        part o.Oscillator{
                part s.Signal{
                        attribute frequency:> Hz;
                        attribute wavelength:> gain = c/o.s.frequency;
                }
        }
}
analysis performance:TradeStudy{…
}
analysis safety:TradeStudy{…
}
```

*Note: these SysML v2 diagrams update the ones used in the March 2023 presentation.

Essential mathematics

- Mathematical functions are single-valued binary relations on a domain
  … $f\,(\mathrm{P_t}, \mathrm{A_e}) = z_1 = \mathrm{P_t}\,\mathrm{A_e}$ is a mathematical function on $X_1 = (\mathrm{P_t}, \mathrm{A_e})$
  … $S(z_2): z_2 \leq 50\ \mathrm{W/m^2}$ is a binary relation[1] in Z_2 ; it is *not* a function
- A *first order model* is an interpretation of language into a relational structure[2]
- ROSETTA is a matrix representation of binary relational structures



ROSETTA will be used to identify, organise, define and store binary relationships; also, to discover and derive binary relationships such as the constraint transformation T.

[2] The interpretation of the relation $S(z_2)$ as a subset of Z_2 *is a model of* $z_2 \leq 50\ \mathrm{W/m^2}$ *mapped into the subset structure of* Z_2.

[1] A binary relation between a variable and a constant is also called a *unary relation*.

Matrix View of ordinary constraints for Safety Analysis:
ROSETTA Framework when objectives are *independent* variables

There are four *defined relations*,

f: $X_1 \rightarrow Z_1$ *and* $R(z_1) \subseteq Z_1$ (performance)

h: $X_2 \rightarrow Z_2$ *and* $S(z_2) \subseteq Z_2$ (safety)

OMG UPR 1.0

$$\begin{array}{ccc} R & \subseteq & Z_1 \\ f^{-1} \downarrow & & \uparrow f \\ V & \subseteq & X_1 \end{array}$$

f: $X_1 \rightarrow Z_1$ *implies an inverse map* f^{-1}: $R \rightarrow V$ *if* $R \cap f\,(X_1) \neq \phi$.

Relational View of Domain Knowledge

N, M, Q

			$R(z_1)$		Z_1
				$S(z_2)$	Z_2
X_1	X_2		Z_1	Z_2	
$V(\underline{x}_1)$		X_1	f		
	$W(\underline{x}_2)$	X_2		h	

The inverse map f^{-1} *is a binary relation that is not necessarily defined on the whole of R. It is only defined on* $R \cap f\,(X_1)$. *Therefore,*

$$V = f^{-1}\,(R \cap f\,(X_1))$$

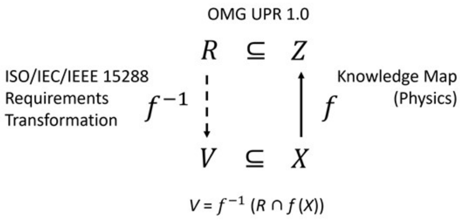
Similarly,

$$W = h^{-1}\,(S \cap h\,(X_2))$$

The map $f\,(z_1) = x_{11}\,x_{12}$ and *instance* $R(z_1)$: $z_1 \leq 50\ \mathrm{Wm^2}$ *define the relation* $V(x_{11}, x_{12})$: $x_{11}x_{12} \leq 50\ \mathrm{Wm^2}$ *in* X_1.

*Design solution and optimisation using constraint driven design**

Requirements Transformation (ISO/IEC/IEEE 15288) using ROSETTA

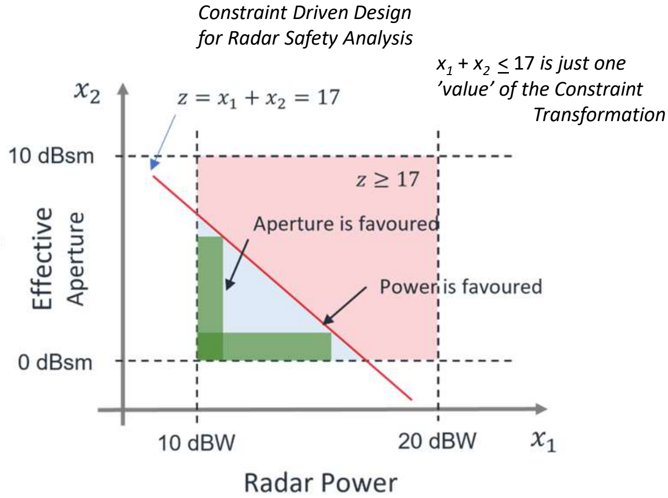


***Representation in the design space $X = (x_1, x_2)$ =* [10 dBW, 20, dBW] x [0 dBsm, 10 dBsm]**
Transformation of requirements (in decibels) defines the solution set $V(x_1, x_2)$, (blue triangle):
Power aperture product $= z = f(x_1, x_2) = x_1 + x_2$

$f^{-1}(R \cap f(X)) = f^{-1}([0, 17] \cap [10, 30])$
$= f^{-1}([10, 17])$

$z \in R = [10, 17] \rightarrow x_1 + x_2$ is a solution

Maximising z (dBWsm) (green rectangles) is constrained by the *safety* requirement.



*Adapted from the Loughborough University WS66 System Design MSc module.

Topics of the Presentation

- Engineering viewpoint of the radar system design and safety analysis
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The ROSETTA framework for radar system design and safety analysis with interrelations is an inherently simple structured MBSE method.

Matrix View of a Relational Structure for Safety Analysis: ROSETTA Framework when the objectives are *dependent* variables

There are three *defined relations*,

$f: X_1 \rightarrow Z_1$

$h: X_2 \rightarrow Z_2$

$g: X_1 \rightarrow X_2$

There is one ***implied relation*** $U(g)$, which is identified using *relational transformation*.

Formulae can be expressed in SysML v2.

$f\,(P_t\,,A_e) = P_t\,A_e$
$h\,(P_t\,,G_t) = P_t\,G_t/4\pi d^2$
$g\,(P_t\,,A_e) = (P_t\,,G(A_e))$
$G(A_e) = 4\pi\,A_e\,/\lambda^2$

Relational View of Domain Knowledge

N, M, Q

				$U(g)$	Z_1
					Z_2
X_1	X_2		Z_1	Z_2	
	g	X_1	f		
		X_2		h	

Relational transformation (of the relation g):

(X_1, X_2) with (X_1, Z_1) and $(X_2, Z_2) \rightarrow \exists U, (Z_1, Z_2) = U(g)$

The new relation *identified* is denoted as U and is *defined* by the chain of relations (Z_1, X_1), (X_1, X_2), and (X_2, Z_2). Thus, U is defined by the composition, $U = h \circ g \circ f^{-1}$.

Solution of the Radar Safety Analysis Problem using ROSETTA (1 of 2)

There are four *defined or implied* relations,

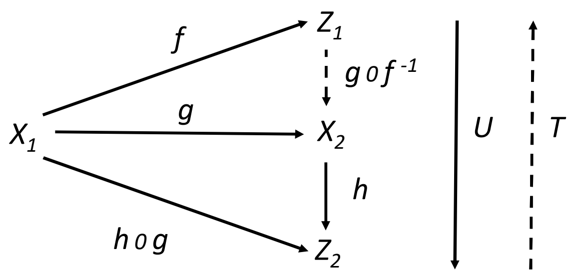
$f: X_1 \rightarrow Z_1$

$h: X_2 \rightarrow Z_2$

$g: X_1 \rightarrow X_2$.

$U(g) = h \circ g \circ f^{-1}$

Relational Transformation: Graphical View



Derivation of T from U(g)

N, M, Q

				$U(g)$	Z_1
			$T=?$		Z_2
X_1	X_2		Z_1	Z_2	
	g	X_1	f		
		X_2		h	

The relational structure of the framework is complete but the constraint transformation T maps $Z_2 \rightarrow Z_1$, not $Z_1 \rightarrow Z_2$. The formal solution for T can be derived from U(g) …

$U = h \circ g \circ f^{-1} \rightarrow T = U^{-1} = (h \circ g \circ f^{-1})^{-1} = f \circ g^{-1} \circ h^{-1}$.

As seen in the algebraic calculations in the appendix,

$z_1 = T\,(z_2) = d^2\lambda^2 z_2$.

Solution of the Radar Safety Analysis Problem using ROSETTA (2 of 2)

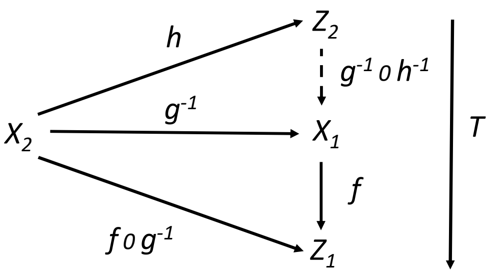
There are three *defined relations*,

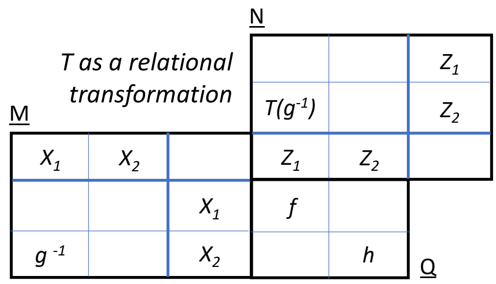
$f: X_1 \rightarrow Z_1$

$h: X_2 \rightarrow Z_2$

$g: X_1 \rightarrow X_2$

Relational Transformation: Graphical View





Relational transformation (of the relation g^{-1}):

(X_2, X_1) with (X_2, Z_2) and $(X_1, Z_1) \rightarrow \exists T, (Z_2, Z_1) = T$

The new relation *identified* is denoted as T, which is *defined* by the chain of relations (Z_2, X_2), (X_2, X_1), and (X_1, Z_1). Thus, T is defined by the composition, $T = f \circ g^{-1} \circ h^{-1}$, as before.

Matrix View of the ROSETTA Framework for Analysis and Design

The design spaces are X_1 and X_2.

The objective spaces are Z_1 and Z_2.

ROSETTA frameworks are used to specify

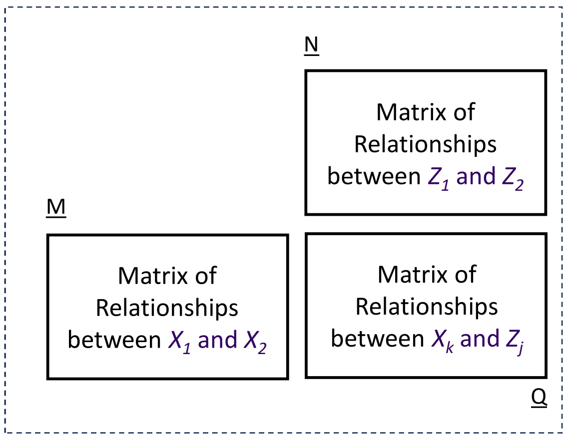
Relational structures for concepts

Interpretation of knowledge about concepts into the relational structures.

This results in a *model of the concept [5]* that **integrates system structure and mathematical expression**.

The structure is complete when *defined relations* have all been identified and all *implied relations* have been defined.

Design solutions are found amongst the various relations: defined, implied or derived (e.g., inverse relations).



The framework depicted is a bottom-up view of analysis and design. By swapping M and N matrices, a top-down view can be represented.

Tool capabilities needed for quantitative modelling and analysis

- The representation and solution of engineering problems using ROSETTA and relational orientation would benefit from tools capable of
  - Integrated matrix and symbolic manipulation of maps
  - Notations to extend the role of algebraic variables to maps as variables and of equations to compositions of maps (algebraic relations)
- This is similar to the type of tools used in category theory such as,

https://www.kestrel.edu/research/specware/

https://www.epatters.org/wiki/algebra/computational-category-theory.html

https://ncatlab.org/nlab/show/TwoVect

Conclusions

Implementation of ROSETTA in SysML v2?

Recap of March 2023 presentation to SE DSIG:
Ordinary constraints in UPR 1.0 can be implemented in SysML v2

Extension of ordinary constraint transform to binary relations → ROSETTA

Need to investigate integration of mathematical tools with SysML v2 tools

Way ahead: SysML v2 extensions to fully address the issue, feeding into SysML v2.1

Ongoing research on extensions of category theory [7] for architecture, analysis and design will be presented in next Mathsig meeting

References

[1] OMG Mathsig Homepage, https://www.omg.org/maths/

[2] OMG, UPR: UML Profile for ROSETTA, v1.0, 2019, https://www.omg.org/spec/UPR

[3] Mavris, Griendling, and Dickerson, Relational-Oriented Systems Engineering and Technology Tradeoff Analysis Framework, *Journal of Aircraft,* vol. 50, no. 5, 2013.

[4] SysML Version 2.0 Release 2023-02, https://github.com/Systems-Modeling/SysML-v2-Release/blob/master/doc/2-OMG_Systems_Modeling_Language.pdf

[5] Dickerson and Ji, *Essential architecture and principles of systems engineering*. CRC Press 2021.

[6] *Systems and Software Engineering – System Life Cycle Processes*, ISO/IEC/IEEE 15288:2015.

Note: ISO/IEC/IEEE 15288:2023 is the latest revision but does not impact this presentation.

[7] Dickerson and Wilkinson, Architecture, Analysis and Design of Systems Using Extensions of Category Theory, *IEEE Open Journal of Systems Engineering*, accepted with minor revisions 20th of April 2024.

C. E. Dickerson, M.K. Wilkinson et al., "Architecture Definition in Complex System Design Using Model Theory," in *IEEE Systems Journal*, vol. 15, no. 2, pp. 1847-1860, June 2021.

The UK Secretary of State, "The Control of Electromagnetic Fields at Work Regulations", Health and Safety Regulation 2016 No. 588.

Questions?

Appendix

A-1 Algebraic expression and solution for the constraint transformation

A-2 Abstract

Architecture, Analysis and Design of Systems Using Extensions of Category Theory, to be published soon in the IEEE Open Journal of Systems Engineering [7]

A-1 Algebra of the Constraint Transformation Relational Structure (in dB)

(1) $z_1 = x_1 + x_2$ — *Power Aperture Product*

(2) $z_2 = x_1 + x_3 - r$ — *Power Density at safety perimeter (d)*

(3) $0 = x_2 - x_3 + k$ — (this derives from $x_3 = x_2 + k$)

$r = 4\pi d^2$	$x_1 = P_t$	in dB	$(x_1 = x_{11} = x_{21} = P_t)$
$k = 4\pi/\lambda^2$	$x_2 = \mathrm{A_e}$	in dB	$(x_2 = x_{12} = \mathrm{A_e})$
	$x_3 = \mathrm{G_t}$	in dB	$(x_3 = x_{22} = \mathrm{G_t})$

$(x_{11}, x_{12}) \in X_1$ and $(x_{21}, x_{22}) \in X_2$ $\quad P_t = x_1 = x_{11} = x_{21}$ is a shared attribute

A-1 (continued) Solution for the Transformation by Algebraic Methods

Add equations (2) and (3) to obtain,

(2) + (3): $z_2 = x_1 + x_2 - r + k$

Substitute (1), $x_1 + x_2 = z_1$, into this to obtain,

$z_2 = z_1 - r + k \rightarrow z_1 = z_2 + r - k$ in dB

In SI units we then have,

$z_1 = (z_2)\,(4\pi d^2)/(4\pi/\lambda^2) = z_2\, d^2\lambda^2$

This defines the constraint transformation as,

$$z_1 = T(z_2) = d^2\lambda^2\, z_2 = 50Wm^2 = 17dBWsm \text{ for } \lambda d = 1.$$

This agrees with both OMG results [2] and a forthcoming paper.

A-2 IEEE paper in final revision for publication

Dickerson and Wilkinson. Architecture, Analysis and Design of Systems Using Extensions of Category Theory [7]

Abstract– The engineering of systems has lacked the scientific and mathematical underpinnings enjoyed by traditional engineering disciplines.

Earlier work of the authors formulated advanced mathematical methods for architecture definition and the use of model theory in engineering. In this paper an extension of the category of relations is defined by a semantically richer algebraic logic, denoted as CoR, suitable for analysis and design. This complements and extends the rigorous basis for our earlier work.

CoR uses category theory coupled with the semantically richer logic as an architectural language for specifying structural views of concepts with concordant graphical and algebraic relational views for analysis and design. A radar case study is used to demonstrate methods for the application of these views to everyday engineering practice. CoR also addresses and can improve the expressiveness of modelling languages such as SysMLv2. Schemata are defined to provide mathematical blueprints of the views.

The intent is that CoR can be implemented in systems tools that are well-suited for engineering practitioners. This work offers a rigorous but practical platform for establishing a new generation of systems engineering methodologies, tools and languages.